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Algebra 1

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Quiz #3

Problems :

1. Let Ω_n be the set of the $n-th$ roots of unity
$$\Omega_n = \{z \in \mathbb{C} : |z| = 0\} = \{e^{2\pi ik/n} : 0 \leqslant k \leqslant n-1\}$$
   (a) Prove that Ω is a group.
   (b) Prove that the map
$$\begin{array}{llll} \phi : & \mathbb{Z}/n\mathbb{Z} & \to & \Omega_n \\ & k & \mapsto & e^{2\pi ik/n} \end{array}$$
   is a well defined isomorphism.
2. (a) Give the definition of a cyclic group G and their classification (i.e. describe G when $|G|$ is finite and infinite).
   (b) Prove that a group with no non-trivial non proper subgroup is cyclic.
3. For $d|n$, give an element of order d of $\mathbb{Z}/n\mathbb{Z}$. Justify your answer.
4. Let X be a set and G a group, given a group action $G \times X \to X$, prove that the stabilizer $Stab(x)$ of a point $x \in X$ is a subgroup of G. Is $G/Stab(x)$ necessarily a group ? To which interesting set is this quotient isomorphic to ?